

Name of Course	:	CBCS(LOCF) B.Sc.(H)Mathematics
Unique Paper Code	:	32351301
Name of Paper	:	BMATH305-Theory of Real Functions
Semester	:	III
Duration	:	3 hours
Maximum Marks	:	75 Marks

Attempt any four questions. All questions carry equal marks.

1. Prove that $\lim_{x \rightarrow -1} \frac{x^2-5}{x^2+7} \neq \frac{-5}{7}$.

Use $\varepsilon - \delta$ definition of limit to prove that $\lim_{x \rightarrow 1} \frac{x^3-3}{x^2+1} = -1$.

Also prove that $\lim_{x \rightarrow 0} x^2 \operatorname{sgn}(x)$ exists. Here sgn denotes the signum function.

2. Let $A \subseteq \mathbb{R}$, functions $f, g: A \rightarrow \mathbb{R}$, c be a cluster point of A and $L \in \mathbb{R}$, $L \neq 0$.

If $\lim_{x \rightarrow c} f = L$ and $\lim_{x \rightarrow c} g = \infty$, then find $\lim_{x \rightarrow c} fg$.

If $\lim_{x \rightarrow c} f = 0$ and $\lim_{x \rightarrow c} g = \infty$ then justify by an example that $\lim_{x \rightarrow c} fg$ need not be infinity.

3. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^2, & \text{if } x \text{ is rational} \\ 5x - 6, & \text{if } x \text{ is irrational} \end{cases}$$

Find all the points at which f is continuous.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be an additive function, that is, $f(a+b) = f(a) + f(b)$ for all $a, b \in \mathbb{R}$. Prove that if f is continuous at some point x_0 , then f is continuous at every point of \mathbb{R} .

4. Show that $f(x) = \frac{x-1}{x+1}$ is uniformly continuous on $[0, \infty)$ and $g(x) = \cos\left(\frac{1}{x}\right)$ is not uniformly continuous on $(0, \infty)$.

If f is continuous on $[0, 2]$ and $f(0) = f(2)$, then prove that there exists $x, y \in [0, 2]$ such that $|y - x| = 1$ and $f(x) = f(y)$.

5. Find the points of relative extrema of the following function on the specified domain

$$f(x) = |x^2 - 25|, \quad -7 \leq x \leq 7.$$

Prove that $ex \leq e^x$, for all $x \in \mathbb{R}$.

Use Mean Value Theorem to find an approximate value of $\sqrt{51}$.

6. If $x \in [0, 1]$ and $n \in \mathbb{N}$, show that

$$\left| \ln(1+x) - \left(x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + (-1)^{n-1} \frac{x^n}{n} \right) \right| < \frac{x^{n+1}}{n+1}.$$

Use this to approximate $\ln(1.5)$ with an error less than 0.01.

Use Taylor's theorem to prove that for all $x > 0$

$$1 + x + \frac{x^2}{2} + \frac{x^3}{3!} < e^x < 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} e^x .$$

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